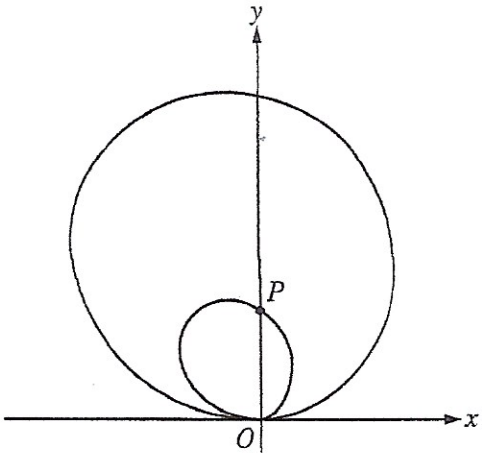
Name: Key

Free Response: Section II, Part A

A graphing calculator is required for problems on this part of the exam.



1. Let r be the function given by $r(\theta) = 3\theta \sin\theta$ for $0 \le \theta \le 2\pi$. The graph of r in polar coordinates consists of two loops, as shown in the figure above. Point P is on the graph of r and the y-axis.

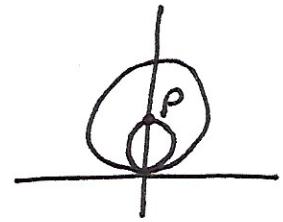
(A) Find the rate of change of the x-coordinate with respect to θ at the point P.

(B) Find the area of the region between the inner and outer loops of the graph.

(C) The function r satisfies $\frac{dr}{d\theta} = 3\sin\theta + 3\theta\cos\theta$. For $0 \le \theta \le 2\pi$, find the value of θ that gives the point on the graph that is farthest from the origin. Justify your answer.

AP Questions

$r(\theta) = 3\theta \sin\theta$ for $0 \le \theta \le 2\pi$



(A) Find the rate of change of X coordinate wrt θ at P

$[0, 2\pi] \times [-15, 10]$

$X = r \cos\theta$

$X = 3\theta \sin\theta \cos\theta$

At Point P, $\theta = \frac{\pi}{2}$

$$\left.\frac{dX}{dt}\right|_{\theta = \pi/2}$$

nDeriv $(3\theta \sin(\theta) \cos(\theta), \theta, \frac{\pi}{2})$

$= -4.712385839$

$= \boxed{-4.712}$

(B) Find area of region between inner and outer loops.

$3\theta \sin\theta = 0$

$\sin\theta = 0$

$\theta = \pi$

outer loop − inner loop

$$\frac{1}{2}\int_{\pi}^{2\pi} (3\theta \sin\theta)^2 \, d\theta - \frac{1}{2}\int_0^{\pi} (3\theta \sin\theta)^2 \, d\theta$$

$\frac{1}{2}(318.497 - 39.441)$

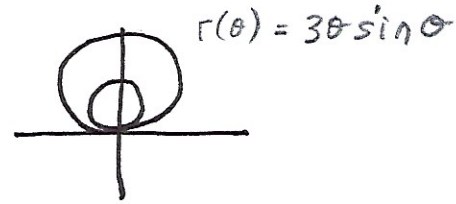
$\boxed{139.528}$

(c) The function r satisfies $\frac{dr}{d\theta} = 3\sin\theta + 3\theta\cos\theta$ for $0 \le \theta \le 2\pi$, find value of θ that gives the point farthest from the origin.

$\boxed{\text{Mode}}$
Polar

$\boxed{\text{Window}}$

θ min	0
θ max	2π
θ step	$\pi/24$
x $[-10, 10]$	y $[-8, 20]$

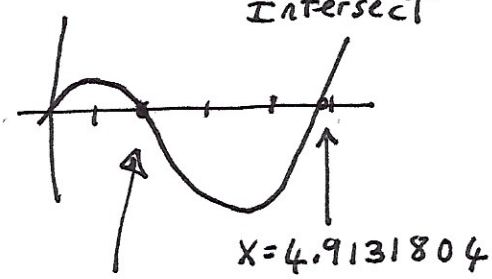


$3\sin\theta + 3\theta\cos\theta = 0$ ← set $\frac{dr}{d\theta} = 0$

$\boxed{\text{mode}}$
function

$\boxed{y=}$
$y_1 = 3\sin X + 3X\cos X$
$y_2 = 0$

$\boxed{2^{nd}}$
Calc
Intersect



$[0, 2\pi] \times [-15, 10]$

$\boxed{\text{mode}}$
Polar

$r = 3\theta\sin\theta$

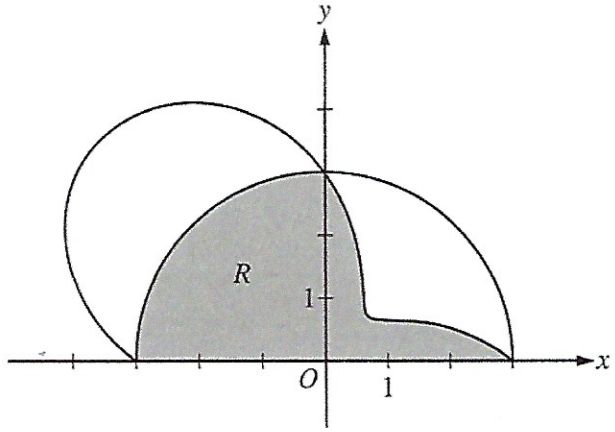
$\boxed{2^{nd}}$
table Set

Indepnt: $\boxed{\text{Ask}}$ ← once table setup is ask

θ	r_1
0	0
2.0288	5.4591
4.9132	-14.44
2π	0

← to see this table go to $\boxed{2^{nd}}$ table and enter values 0, 2.0288, ...

The value of $\theta = 4.913$ gives the point on the graph that is farthest from the origin.



2. The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure above for $0 \le \theta \le \pi$.

(a) Let R be the shaded region that is inside the graph of $r = 3$ and inside the graph of $r = 3 - 2\sin(2\theta)$. Find the area of R.

(b) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$.

(c) The distance between the two curves changes for $0 < \theta < \frac{\pi}{2}$. Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \frac{\pi}{3}$.

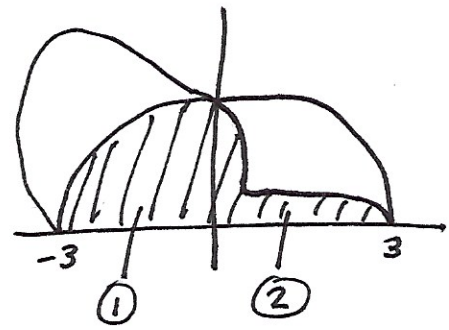
(d) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \ge 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$.

$\boxed{2014}$

graphs of polar curves

$r = 3 \qquad r = 3 - 2\sin(2\theta)$

on $0 \le \theta \le \pi$



a) Find shaded area

$$\text{Area} = \frac{1}{2}\int_a^b [r(\theta)]^2 \, d\theta$$

$r_1 = 3 \qquad r_2 = 3 - 2\sin(2\theta)$

$$\text{Area I} = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(3)^2 = \frac{9}{4}\pi$$

$$\text{Area II} = \frac{1}{2}\int_0^{\frac{\pi}{2}} [3 - 2\sin(2\theta)]^2 \, d\theta$$

$\text{FnInt}((3-2\sin(2\theta))^2, \theta, 0, \pi/2)$

$= 5.278759595$

$\text{Ans}/2 + (9\pi)/4$

$= 9.707963268$

$= \boxed{9.708}$

b) For $r = 3 - 2\sin(2\theta)$, find $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$

$x = r\cos\theta$

$x = (3 - 2\sin(2\theta))\cos\theta$

$$\left.\frac{dx}{d\theta}\right|_{\theta = \pi/6}$$

$\text{nDeriv}((3 - 2\sin(2\theta)) * \cos(\theta), \theta, \pi/6)$

$= -2.366025009$

$= \boxed{-2.366}$

c) Find rate at which distance between two curves is changing wrt θ when $\theta = \frac{\pi}{3}$.

$$D = 3 - (3 - 2\sin(2\theta))$$

$$\left.\frac{dD}{d\theta}\right|_{\theta = \pi/3}$$

nDeriv $(2\sin(2\theta), \theta, \pi/3)$

$= -1.999998667$

$= \boxed{-2}$

d) Particle moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \geq 0$

Find $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$

$r = 3 - 2\sin(2\theta)$

$\frac{dr}{d\theta} = -2\cos(2\theta)2$

$= -4\cos(2 \cdot \frac{\pi}{6})$

$= -4\cos\frac{\pi}{3}$

$= -4(\frac{1}{2})$

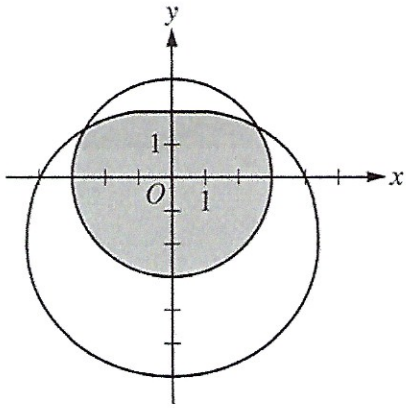
$= -2$

$\frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt}$

$= (-2)(3)$

$= \boxed{-6}$

7.



3. The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

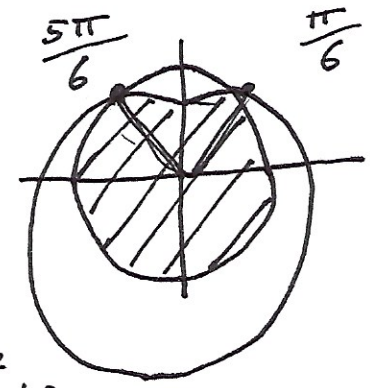
(a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.

(b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.

(c) For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time $t = 1.5$.

$\boxed{2013}$

graphs of $r=3$ and $r=4-2\sin\theta$.
The curves intersect when
$\theta = \pi/6$ and $\theta = 5\pi/6$



a) shaded area

Area of $\frac{2}{3}$ circle

$+ \frac{1}{2}\int_{\pi/6}^{5\pi/6} (4-2\sin\theta)^2\, d\theta$

$= \frac{2}{3}\pi(3)^2$ $\qquad = 5.859174865$

$= 6\pi$

Total Area $= 6\pi + 5.859174865$
$= 24.70873079$
$= \boxed{24.709}$

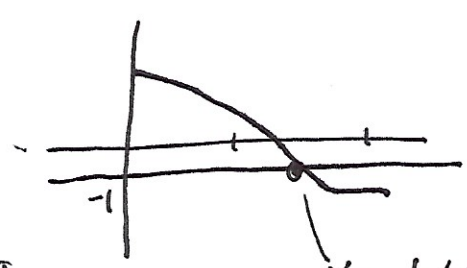
b) Particle moves along polar curve $r = 4-2\sin\theta$
so at time t seconds, $\theta = t^2$. Find t
on $1 \le t \le 2$, so x coordinate position is -1

$$x = r\cos\theta$$
$$x = (4-2\sin\theta)\cos\theta$$
$$-1 = (4-2\sin(t^2))\cos(t^2)$$

$\boxed{\text{Mode}}$
Function

$\boxed{y=}$
$y_1 = (4-2\sin(x^2))\cdot\cos(x^2)$
$y_2 = -1$

$x = \boxed{1.428}$



$x = 1.4279793$

$\boxed{\text{2nd}}$
Calc
5: intersect

c) For Particle described in (b) Find position vector in terms of t.
Find velocity vector at time $t = 1.5$

$y = r \sin\theta$
$y = (4 - 2\sin t^2)\sin t^2$

Position vector $\langle x(t), y(t) \rangle$

$\langle (4 - 2\sin(t^2))\cos(t^2), (4 - 2\sin(t^2))\sin t^2 \rangle$

Velocity at $t = 1.5$ $\langle x'(t), y'(t) \rangle$

$x' = \text{nDeriv}((4 - 2\sin(x^2)) * \cos(x^2), x, 1.5) = -8.072101342$

$y' = \text{nDeriv}((4 - 2\sin(x^2)) * \sin(x^2), x, 1.5) = -1.672933458$

$\boxed{\langle -8.072, -1.673 \rangle}$